Period 1, April 9, 2025

Part A (AB or BC): Graphing calculator required
Scoring Guidelines for Question 1 — 9 points

Learning Objectives: CHA-2.D CHA-3.A CHA-3.C CHA-3.F CHA-4.B LIM-5.A

(a) Use the data in the table to approximate $R'(5)$. Show the computations that lead to your answer. Using correct units, explain the meaning of $R'(5)$ in the context of the problem.

Model Solution	Scoring	
$R'(5) \approx \frac{R(6)-R(4)}{6-4} = \frac{3010-3442}{2} = -216$	Approximation using values from table	1 point 2.B
At time $t=5$ hours (12 P.M.), the rate at which vehicles cross the bridge is decreasing at a rate of approximately 216 vehicles per hour per hour.	Interpretation with units	1 point 3.F 4.B
	Total for part (a)	**2 points**

(b) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\int_0^{12} R(t)\,dt$. Indicate units of measure.

Model Solution	Scoring	
$\int_0^{12} R(t)\,dt \approx 4(R(2)+R(6)+R(10))$	Midpoint sum setup	1 point 1.E
$= 4(3653+3010+1986)$ $= 34{,}596$ vehicles	Approximation using values from the table with units	1 point 2.B 4.B
	Total for part (b)	**2 points**

(c) What is the average number of vehicles crossing the bridge per hour on the weekend day for $0 \le t \le 12$?

Model Solution	Scoring	
$\frac{1}{12-0}\boxed{\int_0^{12} H(t)\,dt} = 2452$ Definite integral Answer	Definite integral	1 point 1.D 4.C
	Answer with supporting work	1 point 1.E
	Total for part (c)	**2 points**

(d) Use $L(t)$ to find the time t, for $12 \le t \le 17$, at which the rate of vehicles crossing the bridge is 2000 vehicles per hour. Show the work that leads to your answer.

Model Solution	Scoring	
$L(t) = H(12) - H'(12)(t-12)$ $H(12) = 2596;\ H'(12) = -216$	Slope	1 point 1.E 4.E
$L(t) = 2000$	$L(t) = 2000$	1 point 1.D
$\Rightarrow t = 14.759$	Answer with supporting work	1 point 1.E 4.E
	Total for part (d)	**3 points**

	Total for Question 1	**9 points**

Part A (AB or BC): Calculator not Permitted
Scoring Guidelines for Question 2 — 9 points

Learning Objectives: FUN-3.B FUN-4.A FUN-5.A FUN-6.D

(a) On what open intervals contained in (0,4) is the graph of f both decreasing and concave down? Give a reason for your answer.

Model Solution	Scoring	
The graph of f is decreasing and concave down on the intervals (1, 1.6) and (3, 3.5)	Answer	**1 point** 2.E
because f' is negative and decreasing on these intervals.	Reason	**1 point** 3.E 4.A
	Total for part (a)	**2 points**

(b) Find the absolute minimum value of f on the interval [0, 4]. Justify your answer.

Model Solution	Scoring	
The graph of f' changes from negative to positive only at $x=2$.	Considers $x=2$ as a candidate	**1 point** 3.B
$f(0)=f(2)+\int_2^0 f'(x)\,dx=f(2)-\int_0^2 f'(x)\,dx=5-(2-6)=9$ $f(2)=5$ $f(4)=f(2)+\int_2^4 f'(x)\,dx=5+(10-14)=1$ On the interval [0, 4], the absolute minimum value of f is $f(4)=1$.	Answer with justification	**1 point** 3.E
	Total for part (b)	**2 points**

(c) Evaluate $\int_0^4 f(x)f'(x)\,dx$.

$$\int_0^4 f(x)f'(x)\,dx=\frac{1}{2}(f(x))^2\Big|_{x=0}^{x=4}$$

$$=\frac{1}{2}\left((f(4))^2-(f(0))^2\right)$$

$$=\frac{1}{2}\left(1^2-9^2\right)=-40$$

Scoring	
Antiderivative of the form $a[f(x)]^2$	**1 point** 1.C
Earned the first point and $a=\frac{1}{2}$	**1 point** 1.E
Answer	**1 point** 2.B
Total for part (c)	**3 points**

(d) Find $g'(2)$. Show the work that leads to your answer.

$g'(x)=3x^2f(x)+x^3f'(x)$

$g'(2)=3\cdot 2^2f(2)+2^3f'(2)=12\cdot 5+8\cdot 0=60$

Scoring	
Product Rule	**1 point** 1.E
Answer	**1 point** 2.B
Total for part (d)	**2 points**

Total for Question 2	**9 points**

Section I: Multiple Choice

45 Questions | 1 Hour 45 minutes | 50% of Exam Score

- Part A: 30 questions; 60 minutes (calculator not permitted).
- Part B: 15 questions; 45 minutes (graphing calculator required).
- Questions include algebraic, exponential, logarithmic, trigonometric, and general types of functions.
- Questions include analytical, graphical, tabular, and verbal types of representations.

Section II: Free Response

6 Questions | 1 Hour 30 Minutes | 50% of Exam Score

- Part A: 2 questions; 30 minutes (graphing calculator required).
- Part B: 4 questions; 60 minutes (calculator not permitted).
- Questions include various types of functions and function representations and a roughly equal mix of procedural and conceptual tasks.
- Questions include at least 2 questions that incorporate a real-world context or scenario into the question.

2007

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	–4	4	2
4	–1	3	6	7

3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by $h(x) = f(g(x)) - 6$.

   (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.

   (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.

   (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t)\,dt$. Find the value of $w'(3)$.

   (d) If g^{-1} is the inverse function of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(x) = f(g(x)) - 6$

$h(1) = F(g(1)) - 6 = F(2) - 6 = 9 - 6 = 3$

$h(3) = F(g(3)) - 6 = F(4) - 6 = -1 - 6 = -7$ IVT

F and g are diff so continuous

$-7 < -5 < 3$

(b) $h(x) = F(g(x))$

$h'(x) = F'(g(x)) \cdot g'(x)$

$h'(1) = F'(g(1)) \cdot g'(1) = F'(2) \cdot g'(1) = 2 \cdot 5 = 10$

$h'(3) = F'(g(3)) \cdot g'(3) = F'(4)\, g'(3) = 3 \cdot 2 = 6$

$h(1) = 3$

$h(3) = -7$

$m = \frac{-7-3}{3-1} = \frac{-10}{2} = -5$

$\frac{h(3) - h(1)}{3 - 1} = -5$ MVT $1 < c < 3$

$h'(c) = -5$

(c) $w(x) = \int_1^{g(x)} F(T)\,dT$

$w'(x) = F(g(x)) \cdot g'(x)$

$w'(3) = F(g(3)) \cdot g'(3) = F(4) \cdot g'(3) = -1 \cdot 2 = -2$

$\int_1^{g(x)} T^2\, dT = \frac{1}{3}T^3 \Big[_1^{g(x)}$

$w(x) = \frac{1}{3}(g(x))^3 - \frac{1}{3}(1)^3$

$w'(x) = \cancel{\frac{1}{3}}(g(x))^2 \cdot g'(x) - 0$

(d) $g^{-1}(x)$ at $x=2$ Tangent Line

$g(?)=2 \Rightarrow g(1)=2$

$g^{-1}(2) = ? = 1$ Point $(2,1)$

$g^{-1\prime}(x) = \dfrac{1}{g'(g^{-1}(2))} = \dfrac{1}{g'(1)} = \frac{1}{5}$

$m = \frac{1}{5}$ Point $= (2,1)$

$y - 1 = \frac{1}{5}(x-2)$

2016

t (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by $W(t) = 2000e^{-t^2/20}$ liters per hour for $0 \le t \le 8$, where t is measured in hours. Water is removed from the tank at a rate modeled by $R(t)$ liters per hour, where R is differentiable and decreasing on $0 \le t \le 8$. Selected values of $R(t)$ are shown in the table above. At time $t = 0$, there are 50,000 liters of water in the tank.

(d) For $0 \le t \le 8$, is there a time t when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.

into Tank Rate $= 2000e^{-T^2/20} \Rightarrow$ $w(8) = 81.52$, $w(0) = 2000$ } IVT

out of Tank $R(T)$

diff

Same Rate $R(t) - w(T) = 0$

$w(0) - R(0)$

$2000 - 1340 = 660$

$w(8) - R(8)$

$81.52 - 700 = -618.48$

Pass 0 IVT

Same Rate $R(T) = w(T)$

2019

Tells concavity

6. Functions f, g, and h are twice-differentiable functions with $g(2) = h(2) = 4$. The line $y = 4 + \frac{2}{3}(x - 2)$ is tangent to both the graph of g at $x = 2$ and the graph of h at $x = 2$.

continious

$g'(2) = \frac{2}{3}$

(a) Find $h'(2)$. = $h'(2) = \frac{2}{3}$

(b) Let a be the function given by $a(x) = 3x^3 h(x)$. Write an expression for $a'(x)$. Find $a'(2)$.

(c) The function h satisfies $h(x) = \dfrac{x^2 - 4}{1 - (f(x))^3}$ for $x \neq 2$. It is known that $\lim_{x \to 2} h(x)$ can be evaluated using L'Hospital's Rule. Use $\lim_{x \to 2} h(x)$ to find $f(2)$ and $f'(2)$. Show the work that leads to your answers.

(d) It is known that $g(x) \le h(x)$ for $1 < x < 3$. Let k be a function satisfying $g(x) \le k(x) \le h(x)$ for $1 < x < 3$. Is k continuous at $x = 2$? Justify your answer.

Squeeze Theorem

$h(2) = k(2) = 4 = g(2)$

(b) $a(x) = 3x^3 \, h(x)$

$a'(x) = 9x^2 \cdot h(x) + 3x^3 \cdot h'(x)$

$a'(2) = 9(2)^2 \cdot h(2) + 3(2)^3 \cdot h'(2) = 9 \cdot 4 \cdot 4 + 3 \cdot 8 \cdot \frac{2}{3} =$

(c) $h(x) = \dfrac{x^2 - 4}{1 - (F(x))^3}$

$h(2) = 4$

$h(x)$ is continious

$\frac{0}{0}$ or $\frac{\infty}{\infty}$ or $\frac{\phi}{\phi}$

$4 = \lim_{x \to 2} \dfrac{x^2 - 4}{1 - (F(x))^3} = \dfrac{2^2 - 4}{1 - (F(2))^3} = \dfrac{0}{0}$

$\lim_{x \to 2} \dfrac{2x}{0 - 3(F(x))^2 \cdot F'(x)} = 4$

$1 - (F(2))^3 = 0$

$F(2) = 1$

$\dfrac{2 \cdot 2}{-3(F(2))^2 F'(2)} = 4$

$\cancel{-3F'(2)} \cdot \dfrac{4}{\cancel{-3 \cdot 1 \cdot F'(2)}} = 4 \cdot -3F'(2)$

$\dfrac{4}{-12} = \dfrac{-12 F'(2)}{-12}$

$-\frac{1}{3} = F'(2)$

8. Let f be a function with first derivative defined by $f'(x) = \frac{2x^2 - 5}{x^2}$ for $x > 0$. It is known that $f(1) = 7$ and $f(5) = 11$. What value of x in the open interval $(1,5)$ satisfies the conclusion of the Mean Value Theorem for f on the closed interval $[1,5]$?

(A) 1 (B) $\sqrt{\frac{5}{2}}$ (C) $\sqrt[3]{10}$ (D) $\sqrt{5}$

MVT

$\frac{11-7}{5-1} = \frac{4}{4} = 1$

$F'(x) = \frac{2x^2-5}{x^2} = 1$ MVT

$\cancel{x^2} \cdot \frac{2x^2-5}{\cancel{x^2}} = 1 \cdot x^2$

$2x^2 - \cancel{5} = \cancel{x^2}$

$-x^2 + \cancel{5} \quad -\cancel{x^2} \quad +5$

$x^2 = 5$

$x = \sqrt{5}$

20011B (calc)

1. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

$M'(T) = \frac{1}{400}(9T^2 - 60T + 330)$

$M'(T) = S'(T)$

$M'(T) - S'(T) = 0$

$M'(0) = .825$

$S'(0) = 1.5$

$M'(15) = 3.64$

$S'(15) = 2.37$

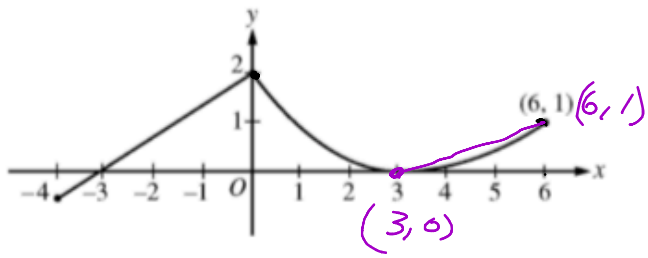
$M'(0) - S'(0) = .825 - 1.5 = -.675$

$M'(15) - S'(15) = 3.64 - 2.37 = 1.27$

$M'(T) - S'(T) = 0$ must happen

$0 \le T \le 15$ IVT

2009B



Graph of f

3. A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$. concave UP

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

$$\frac{F(3)-F(6)}{3-6} = \frac{0-1}{-3} = \frac{1}{3}$$

2. If $f(x) = \sin\left(\frac{x}{2}\right)$, then there exists a number c in the interval $\frac{\pi}{2} < x < \frac{3\pi}{2}$ that satisfies the conclusion of the Mean Value Theorem. Which of the following could be c?

(A) $\frac{2\pi}{3}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{6}$

(D) π

(E) $\frac{3\pi}{2}$

$$\frac{\sin\frac{\pi}{4} - \sin\frac{3\pi}{4}}{\frac{\pi}{2} - \frac{3\pi}{2}} = \frac{\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}}{-\pi} = 0$$

$$f'(x) = \left(\cos\left(\frac{x}{2}\right)\right) \cdot \frac{1}{2} = 0$$

$$\cos\frac{\pi}{2} = 0$$

$$x = \pi$$

7.

x	0	2	5	9	11
$g(x)$	1	2.8	1.7	1	3.4

crosses

The table above shows selcted values of a continuous function g. For $0 \le x \le 11$, what is the fewest possible number of times $g(x) = 2$?

(A) One

(B) Two

(C) Three

(D) Four

11. Let f be the function defined by $f(x) = x + \ln x$. What is the value of c for which the instantaneous rate of change of f at $x = c$ is the same as the average rate of change of f over [1, 4]?

a. 0.456

b. 1.244

c. 2.164

d. 2.342

e. 2.452

$$f'(x) = 1 + \frac{1}{x}$$

$$\frac{f(1) - f(4)}{1 - 4} =$$

12. Let f be a function that is differentiable on the open interval (1, 10). If $f(2) = -5$, $f(5) = 5$, and $f(9) = -5$, which of the following must be true?

I. f has at least 2 zeros.
II. The graph of f has at least one horizontal tangent.
III. For some c, $2 < c < 5$, $f(c) = 3$.

a. None b. I only c. I and II only d. I and III only e. I, II, III